Week Three – Linear Graphs

Warm-Up

Section A

Work out the gradient and the y-intercept of these lines.

a $y=7x-4$ **b** $y+2x=3$ **c** $x-y=4$ **d** $3x+2y=7$

e $5x-2y=9$ **f** $5y-3x=0$ **g** $x+6y+3=0$ **h** $3(y-2)=4(x-1)$

Section B

Find the gradient of the line through each pair of points.

a (3, 7) and (2, 8) **b** (5, 2) and (–4, –6) **c** (1.3, 4.7) and (2.6, –3.1)

d $\left(\frac{1}{2},\frac{1}{3}\right)$ and $\left(\frac{3}{4},\frac{2}{3}\right)$ **e** $(\sqrt{3}, 2)$ and $\left(2\sqrt{3}, 5\right)$ **f** $(3a, a)$ and $(a, 5a)$

Section C

Find the equation of the line through each pair of points.

a (2, 5) and (0, 6) **b** (1, –3) and (2, –5) **c** (4, 4) and (7, –7)

d (8, –2) and (4, –3) **e** (–3, –7) and (5, 9) **f** $(\sqrt{2}, -\sqrt{2})$ and $\left(3\sqrt{2}, 4\sqrt{2}\right)$

Perpendicular Lines

Example 1 LINK

Decide whether or not each line is parallel or perpendicular to the line $y = 4x - 1$

a $2x + 8y = 5$ **b** $20x + 5y = 2$ **c** $16x - 4y = 5$

$m_1 = 4$

Perpendicular: $m_1 \times m_2 = -1$

a) $2x + 8y = 5$

$8y = 5 - 2x$

$y = \frac{5}{8} - \frac{1}{4}x$

$m_2 = -\frac{1}{4}$

$m_1 \times m_2 = 4 \times -\frac{1}{4}$

$= -1$

They are perp.

b) $20x + 5y = 2$

$5y = 2 - 20x$

$y = \frac{2}{5} - 4x$

$m_2 = -4$

$m_1 \times m_2 = -4 \times 4$

$= -16$

Not parallel or perp.

c) $16x - 4y = 5$

$16x - 5 = 4y$

$4x - \frac{5}{4} = y$

$m_2 = 4$

$m_1 = m_2 = 4$

$\therefore$ parallel.

Example 2 LINK

The line l_1 has equation $7x+4y=8$. The line l_2 is perpendicular to l_1 and passes through the point $(7, 3)$. Find the equation of l_2 in the form $ax+by+c=0$ where a, b and c are integers.

$7x + 4y = 8$

$4y = 8 - 7x$

$y = 2 - \frac{7}{4}x$

$m = -\frac{7}{4}$ $\therefore$ $m_{perp} = \frac{4}{7}$

$y = mx + c$

$y = \frac{4}{7}x + c$

Using $(7, 3)$

$3 = \frac{4}{7}(7) + c$

$3 = 4 + c$

$c = -1$

$y = \frac{4}{7}x - 1$

$7y = 4x - 7$

$0 = 4x - 7y - 7$

$4x - 7y - 7 = 0$

Example 3 LINK

Find the equation of the perpendicular bisector of the line segment joining (3, –4) and (9, –6)

Midpoint: $\left(\frac{3+9}{2}, \frac{-4+-6}{2}\right)$

$= (6, -5)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-6 - -4}{9 - 3}$

$= \frac{-2}{6}$

$= -\frac{1}{3}$

$m_{perp} = 3$

$y = mx + c$

$y = 3x + c$

Using $(6, -5)$,

$-5 = 3(6) + c$

$-5 = 18 + c$

$-23 = c$

$y = 3x - 23$

Exercise 1

Which of these lines is either parallel or perpendicular to the line with equation $y=6x+5$?

a $2x+12y+3=0$ **b** $18x+3y=2$ **c** $3x-\frac{1}{2}y+5=0$

Which of these lines is either parallel or perpendicular to the line with equation $y=\frac{2}{3}x-4$?

a $24x+16y+3=0$ **b** $6x+9y+2=0$ **c** $2x-3y=7$

Which of these lines is either parallel or perpendicular to the line with equation $6x+12y=1$?

a $2y=5-x$ **b** $9x=18y+4$ **c** $10x-5y+3=0$

Exercise 2

Give your answers in the form $ay + bx + c = 0$.

The line l_1 has equation $y=5x+1$

a Find the equation of the line l_2 which is parallel to l_1 and passes through the point $(3, -3)$

b Find the equation of the line l_2 which is perpendicular to l_1 and passes through the point $(-4, 1)$

The line l_1 has equation $y=3+\frac{1}{2}x$

a Find the equation of the line l_2 which is parallel to l_1 and passes through the point $(-1, 5)$

b Find the equation of the line l_2 which is perpendicular to l_1 and passes through the point $(6, 2)$

The line l_1 has equation $3x+y=9$

a Find the equation of the line l_2 which is parallel to l_1 and passes through the point $(8, -2)$

b Find the equation of the line l_2 which is perpendicular to l_1 and passes through the point $(-1, -1)$

Exercise 3

Give your answers in the form $ay + bx + c = 0$.

Find the equation of the perpendicular bisector of the line segment joining each pair of points.

a $(5, -7)$ and $(-3, 5)$ **b** $(-5, -9)$ and $(5, 5)$ **c** $(-6, 2)$ and $(4, 12)$

d $(2, -7)$ and $(-1, 2)$ **e** $(-13, -5)$ and $(15, -12)$